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Paola Azar* y Gabriela Sicilia**

Resumen

Este trabajo evalúa las potencialidades de mejora en los resultados académicos de alumnos de escuelas públicas primarias de Uruguay. Utilizando datos a nivel de estudiante generados por la primera evaluación nacional de logros educativos propone un análisis de meta-fronteras multinivel siguiendo a Silva-Portela y Thannassoulis (2002). Se encuentra que, en promedio, los resultados académicos podrían mejorar 19.2% y que ello depende, principalmente de las condiciones individuales de los estudiantes. La incapacidad de las escuelas para transformar el potencial de sus estudiantes en resultados concretos se debe a una inadecuada dotación de recursos y no a problemas de gestión. Las restricciones a nivel de centro impactan sobre todo a los alumnos de contextos socioeconómicos más deprimidos y a aquellos que tienen resultados académicos más bajos.

Palabras clave: desempeño educativo, enfoque metafrontera multinivel, educación primaria

Código JEL: C61, H52, I21

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Abstract

This paper discusses the potential improvements in pupil's academic results at public primary schools in Uruguay. Using student level data from the first national assessment of educational achievements, we decompose education attainments into pupil's own effort and school value added following a multilevel metafrontier approach originally introduced by Silva-Portela and Thannassoulis (2002). We find that on average, pupils miss 19.2% of their potential achievement, mainly driven by their own under-performance. The extent of output students cannot obtain because of school effects is mainly explained by suboptimal resource availability at the school level rather than schools' own managerial ability. The shortfall in the school's contribution to efficiency affects those students in the least advantaged socioeconomic contexts and those with lower test scores.

Keywords: educational performance, multilevel metafrontier approach, primary education

JEL Classification: C61, H52, I21

1. Introduction

Over the last years, the need to improve the performance of students and their learning opportunities in the context of tight budget constraints has turned efficiency into a powerful argument. The efficiency approach widens the natural policy focus on schooling funding to include concerns on the extent to which this funding is directed to areas that best influence teaching and learning outcomes (Woessman, 2008; OCDE, 2017; Izquierdo et al., 2018).

Following Debreu (1951) and Farrell (1957), a measure of technical efficiency is based on the comparison between the maximum potential output and the actual outputs, given a combination of available inputs. When applied to the schooling system, this measure may help pupils achieve the best academic performance (i.e. education output) they are believed to be capable.

However, useful as they are, efficiency assessments in education are more difficult to interpret than in other areas. The main reason is that the pupil performance is not the result of a standardized production process. Schooling outcomes depend on teaching and learning, educational authorities' decisions, the school context, the family and personal background of students. Moreover, each of these aspects feed into one another. These conditions turn the task of shoving pupils' performance to the best attainable into a complex challenge. Hence, one contribution to improve policy designs is to identify the extent to which public interventions should focus on pupils or school practices.

In this paper, we conduct an efficiency assessment which separates student's and school's responsibilities for education attainments focused on public primary schooling in Uruguay. The country provides an interesting setting to study this question. Though being one of the most developed economies in Latin America, the Uruguayan schooling system suffers from a persistent resource endowment restriction. After a dynamic expansion between 2005 and 2012, public primary education spending has remained stagnant around 1,1% of GDP, below the figure for the OCDE and many Latin American countries (DINEM, 2020). Besides, the performance of primary schooling pupils does not progress in line with expectations. According to regional academic tests carried out by UNESCO, Uruguayan students are well positioned among the top achievers, but they have not improved their results comparing 2006 and 2013 (INEEd, 2015a). Finally, the country presents one of the strongest associations between socioeconomic conditions and performance in international comparisons, both at the primary and secondary education cycles (INEEd, 2015 and 2019).

In this context, it is important to learn whether students and schools in the Uruguayan system are achieving their full potential and the extent to which these results can be enhanced by acting on their current unexploited "production capacity". Additionally, the focus on primary school pupils let us drive the attention to the earlier years of the life cycle, when effective resource allocation would be more likely to improve future academic performance (Cunha et al., 2006; Heckman, 2006; Woessman, 2008).

Our quantitative analysis uses a recently created database (ARISTAS) containing information about pupils and their academic results based on a national assessment which first results were issued in 2017. Unlike international tests, this source takes into account the academic contents and learning goals emphasized at the national level. In this paper, we focus on public primary pupils from the 6th grade (the last year of the primary cycle). The emphasis on the public system allows us to cover more than 80% of primary enrolment distributed across distinct school types characterized by a different length of the schooling day (and consequently different resource endowments). Likewise, we avoid any selection bias that may arise from the fact that families with higher socio-economic status may self-select into private schools.

In order to disentangle school from pupil impacts on efficiency we revisit the original proposal by Silva-Portela and Thanassoulis (2001) and Thanassoulis and Silva-Portela (2002) and build on it based on Thieme et al. (2013). Accordingly, we apply a metafrontier approach which measures the efficiency of units in relation to separate best practice frontiers (Battese et al. 2004; O'Donnell, et al., 2008). We estimate one local frontier corresponding to students within their school and an overall frontier comprising the best practice pupils among all schools analyzed. We interpret the distance to the local frontier as depending on the student's effort as she is compared with the output levels reached by similar counterparts. The distance to the overall frontier reflects the extent to which the school is unable to turn the potential of its pupils into the outputs attained by the best performing schools in the sample.

To better understand the relative effectiveness of schools in relation to the pupil's performance, we resort to multilevel models and compute the overall frontier by including different school level inputs (including the school environment). Efficiency scores are computed through the order-m partial frontier approach (Cazals et al., 2002). The technique provides results which are robust to atypical observations and which do not suffer from the "curse of dimensionality", by assuring the same size for the reference set (Daraio and Simar, 2007).

Our results are quite informative about the sources of potential inefficiencies. We find that on average, pupils miss 19.2% of their potential achievement. This under-achievement is mainly driven by the individual under-performance. This finding remains stable if students are aggregated according to socioeconomic level and test performance as well as across different types of public schools. The contribution of schools to pupils' under-attainment mostly depends on inadequate resource availability at the school level. Issues around the context in which the school operates are not so relevant. However, different from the individual component, we do find variations: inefficiencies attributable to schools mainly affect those in the least advantaged socioeconomic contexts and with the lowest test scores. Finally, within different school-types we identify that schools tend to be more effective with the more dedicated students. These conclusions underline that resource restrictions together with school inefficiencies lay behind the difficulties of the system to help the more disadvantaged students make up for their original drawbacks.

The analysis provides new evidence for an unexplored subject in Uruguay. Previous research on education efficiency is limited in the country. Besides, it refers to secondary education with records of the Programme for International Student Assessment- PISA (Sicilia, 2014; Santin and Sicilia, 2015; Azar et al., 2018). Instead, some authors look into the effects of primary schools but again they are concerned with the student's attainment in secondary education. Hence, based on parametric methodologies Cardozo et al. (2017) show no significantly differing influence of Full Time schools compared to others on the subsequent academic path of the students. However, Da Rocha et al. (2011) and Lado (2019) do find that the type of primary school affects the academic performance in the next schooling cycle. Furthermore, De Melo and Machado (2018) suggest that it influences the likelihood of dropout of lower-secondary students.

Beyond the specific country data, this paper follows a strand of works engaged on a fair assessment of student's and school's responsibility for the efficiency performance. Most of the literature in the field of education efficiency relies upon school level data (De Witte and López-Torres, 2015). Conversely, our research is related to a reduced number of studies which applies non-parametric techniques making use of pupil level data within a hierarchical structure (that is, regarding that students perform within schools). They generally follow from the proposal of Silva-Portela and Thanassoulis (2001) and Thanassoulis and Silva-Portela (2002) and find higher inefficiencies attributable to pupils than to schools. In the case of these authors, they compute school and pooled frontiers (metafrontiers) to separate individual and institutional contributions to the overall efficiency for a sample of 18- aged students in England. Some years later, De Witte et al. (2010), Portela and Camanho (2010) and Portela et al. (2013) replicate the analysis for British and Portuguese secondary schools, respectively. They argue that the school efficiency effect might be assimilated to the concept of "school value added", just as in the Multilevel Linear Model setting. In the case of Portela et al. (2013) they also extend the analysis to consider changes over time.

Other papers use an extension of the metafrontier framework to include multi-level models. In Cordero et al. (2016) they apply this methodology to discuss the differences between public and subsidized schools in Spain whereas Thieme et al. (2013) build partial frontiers by including different student and school level inputs. Their study considers a sample of Chilean primary school students in their 4th year and it is the most closely related to ours.

The present paper is organized as follows. The key features of the educational system under analysis are presented in Section 2. Next, we describe the methodology and in Section 4 the data and empirical approach. We discuss results in Section 5 and Section 6 concludes.

2. Public primary education system in Uruguay

Compulsory education in Uruguay starts when children are 4 and finishes at upper secondary school, when students are 17. The system is highly centralized: all decisions are taken by a central governing council (Consejo Directivo Central, CODICEN) which co-ordinates the activities of 4 education councils related to each of the schooling levels (pre-primary and primary, secondary, technical and teaching training). Each of these councils is in charge of managing the teaching and non-teaching staff and the financial resources in the public sphere. They also provide the guidelines for pedagogical practices at public schools and directives to monitor the curricula at both public and private institutions.

The primary schooling system has a universal coverage for the children between 6 and 11 years old, no matter their socioeconomic origin. In fact, it has been around 99% since the mid-1950s (INEEd, 2018b). In this context, the share of primary students attending public schools is above 83% (INEEd, 2017). Thus, most of primary school students belongs to the system funded by tax payers and organized according to the directives of the CODICEN.

The public schools are divided into 6 types: Regular, Practice, "Learning", Extended Time, Full Time and Rural schools. They all share the same teaching curricula and organizing rules but differ in the length of the school day and the material and personal resources assigned to them. School hours are 4 at Regular, Practice and Learning schools. However, Practice schools receive those students about to complete their teaching degree and therefore their teaching staff is particularly highly qualified. In turn, Learning schools are situated in the most disadvantaged socioeconomic contexts. For this reason, together with the regular classroom instruction they implement education programs to deal with specific pedagogical, psychological or family needs aiming to lift student achievements.

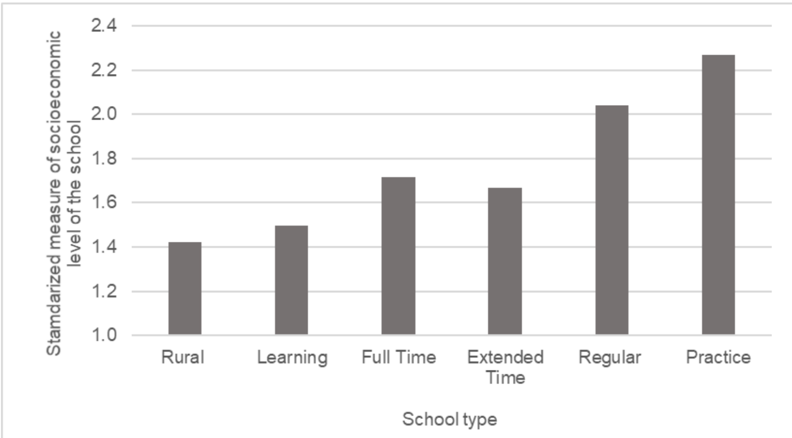
The case of Extended Time schools is similar to the Regular schools but they provide some extra-curricular activities (arts, sports, computing) which make pupils stay for 3 additional hours. Full time schools are double shift schools (students stay a total of 7 and a half hours) and finally the school day in Rural schools lasts 5 hours (ANEP, 2019). Rural schools make up 5% of public primary enrolment. For the other cases, the enrolment distribution is as follows: Learning and Regular schools 31%, Full Time and Practice schools 17% and Extended Time schools 3% (ANEP, 2019).

Comparatively, resource endowments at Full Time and Learning schools imply a greater overall public education spending than in the rest of schools. However, this extended amount does not entail that teachers receive particularly higher salaries. Considering Regular and Learning schools, teachers are paid a small extra-compensation in problematic contexts: the extra-pay represents 10% of the salary for a teacher with no experience and 6% of the wage for a teacher with 25 years of experience. For teachers at Full Time schools, who work a double shift at the same institution, the pay is below the double of the wage paid at a Regular school (INEEd, 2016).

Families can freely choose the public school for their children. In case the demand exceeds the school capacity, the rule to admit students considers whether the families live in the neighborhood of the school, whether any of the parents or guardians work in that area or the presence of siblings already enrolled at the school. Full-time schools also take into account the child's household income and the labor market situation of the child's mother (Santiago et al. 2016).

Even though no particular restriction applies, students are not really randomly distributed across the public school system. The students typically attend their neighborhood school. This leads to a considerable socio-spatial segregation: the distribution of students is highly homogenous within schools but it differs among them, mostly because of the socioeconomic conditions of the district in which the schools are placed. In the public system, almost 50% of public schools belong to a disadvantaged or very disadvantaged context according to the socioeconomic characteristics of their students. Figure 1 summarizes the average differences in the socioeconomic contexts by school type for students in the 6th grade of primary education (at the end of the schooling cycle). The measures are based on a scale which ranges between 1 and 4, going from very poor to excellent socioeconomic contexts (INEEd, 2018a).

Figure 1 Average socioeconomic status of primary schools based on 6th grade students



Source: own computation based on ARISTAS database

According to Figure 1, even within the public system, schools show some differences in relation to the population they receive. Rural and Learning schools are situated in the poorer contexts while Regular and, particularly, Practice schools are in more advantaged environments. Full Time schools receive students from highly disperse socioeconomic contexts. As a comparison, at private schools, largely situated in favorable or very favorable contexts, the indicator reaches, on average, to 3.

Schools are autonomous to decide how to group students into the classrooms. While schools may group students taking into account some special needs, the distribution does not generally rely on the abilities of students (INEEd, 2015a).

Regarding academic achievements, Uruguay performs well in the Latin American context. The country shows a larger proportion of students among the top achievers and a lower share among the bottom performers than the regional average, according to the Third Regional Comparative and Exploratory Study (INEED, 2015b). However, Uruguay exhibited one of the widest achievement gaps between the 10th and 90th percentile and one of the most important urban-rural division of the region in mathematics (UNESCO, 2008). Accordingly, primary students report the highest academic achievements in favorable contexts. These results are consistent both for reading and mathematics assessments as reflected in the national performance assessment called ARISTAS in 2017. Figure A-1 in the Appendix shows a positive correlation of socioeconomic conditions and performance for the public schooling system.

3. Methodology

The theoretical approach used in this paper for linking input resources to educational outcomes at the student level is based on the well-known educational production function proposed by Levin (1974), Hanushek (1979) and Hanushek et al (2013), considering the possible existence of inefficient behaviors:

$$y_i = f(x_i) \cdot u_i \qquad (1)$$

where $y \in R_+^q$ represents the educational output vector for student i usually represented by test scores in a standardized assessment, $x \in R_+^p$ represents the vector of inputs including the students own abilities, parental and socioeconomic background and school educational resources, and $u_i$denotes the technical efficiency level. Values of $u_i = 1$ imply that the evaluated student is fully efficient, meaning that given the initial input endowment and the available technology; she is maximizing her outputs. Values of $u_i > 1$ indicate that the student is inefficient and therefore the efficiency rate, $\theta_i = 1/u_i$ indicates the amount by which the actual output vector should be multiplied to reach the frontier in which case the student would be fully efficient.

The measurement of the efficiency component is associated with Farrel's concept of technical efficiency (Farrell 1957). Farrell defines the production frontier as the maximum level of output that a decision-making unit (DMU) can achieve given its inputs and the technology (output orientation). In practice, the true production frontier and the technology is not known and should be estimated from the relative best practices (students in our case) observed in the sample.

There are two main groups of techniques for estimating the production frontier: parametric, or econometric approaches, and non-parametric methods based on mathematical optimization models. Although the use of parametric approaches has increased in education in the last decades (De Witte and López-Torres, 2015), nonparametric methods have been the most extensively applied when measuring educational efficiency. The two most well-known non-parametric techniques for estimating efficiency are Data Envelopment Analysis (Charnes et al. 1978, Banker et al. 1984) and Free Disposal Hull (Deprins, et al. 1984). They account for multiple outputs and inputs while they do not require any *a priori* assumption on the functional form of the production process. Just some general microeconomic properties for production functions are assumed (Shephard, 1970; Daraio and Simar, 2007). Both techniques draw the production frontier connecting efficient units. However, while DEA builds up the production frontier through a convex piecewise linear combination of best performers, the FDH technique is even more flexible, because it relaxes the convexity assumption and efficiency performance can be evaluated on actual best practice units (students).

In this research we choose an FDH approach because it ensures that all reference units are real pupils. We apply a robust order-m version of the FDH model, which allows us to mitigate the influence of outliers and the curse of dimensionality. Finally, to decompose the overall student efficiency into the contribution of pupils' own effort and school value added, we adopt the metafrontier framework rooted on Silva-Portela and Thannassoulis

(2002) complemented with a multilevel model (Thieme et al., 2013). The following parts of this section are devoted to explain the FDH model and its robust version as well as the main features of the multi-level metafrontier framework.

3.1 The FDH model

The measurement of student's efficiency using nonparametric techniques lies on the estimation of the relative performance of each unit (student) to the boundary of the production possibility set $P(x) = \{(x, y) \in \Re_+^{p+q} | x \text{ can produce } y\}$. P(x) includes all feasible combinations of inputs and outputs but since it is unobserved, it has to be estimated from the best practices in the observed sample.

The FDH assumes that all observed input-output combinations are feasible and a free disposability in inputs and outputs for de production possibility set P(x). This last assumption means that if (x, y) ∈ P(x) then (x', y') ∈ P(x) for any x' ≥ x and y' ≤ y. In our context, this implies that a student might obtain a lower level of achievement than it would be expected from best practices (the frontier) given her level of inputs. Based upon these assumptions it is possible to estimate the boundary of P(x) as:

$$\hat{P}(x)_{FDH} = \{(x, y) \in \Re_+^{p+q} | y \leq y_i, x \geq x_i, i = 1, \dots, n\} \qquad (2)$$

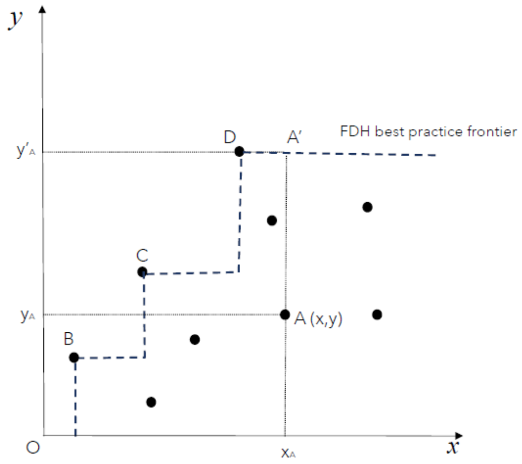
where y_i and x_i corresponds to the output and input vectors of student i, respectively. The students who are undominated are identified as the best practices. Then, the best practice frontier enveloping all the students is therefore characterized by a step- wise function. We illustrate these ideas in Figure 2 for a single-input single-output case. In this simple example, students B, C and D are the best practices (reference units) and the rest of students are identified as inefficient. The output-oriented efficiency measure for each student is then computed as the ratio between the potential maximum output she can achieve relative to her actual achievement (in our example the distance Oy'_A/Oy_A)

In general, the output-oriented FDH efficiency score $\hat{\lambda}_{FDH}$ for each student can be obtained by solving the following mixed integer linear programming problem:

$$\hat{\lambda}_{FDH} = \max\left\{\lambda \middle| \lambda y \leq \sum_{i=1}^{N} \gamma_i y_i ; x \geq \sum_{i=1}^{N} \gamma_i x_i ; \sum_{i=1}^{N} \gamma_i = 1; \gamma_i \in \{0,1\}; i = 1, \dots, n\right\} \qquad (3)$$

where $\hat{\lambda}_{FDH} \geq 1$ is the efficiency score, is the output vector (q×1) and is the input vector $(p \times 1)$. The $(n \times 1)$ vector γ contains the virtual weights of each student determined by the problem solution. When $\hat{\lambda}_{FDH} = 1$ the evaluated student belongs to the frontier (is fully efficient), whereas $\hat{\lambda}_{FDH} > 1$ indicates that the student is inefficient. The value of $\hat{\lambda}_{FDH} > 1$ indicates the equi-proportional expansion over all outputs needed to reach the frontier. Therefore, the higher the score value $\hat{\lambda}_{FDH} > 1$, the greater the inefficiency level.

Figure 2 FDH frontier



The original nonparametric FDH approach presents some significant shortcomings that should be taken into account when estimating efficiency measures of school performance. Firstly, statistical inference is not possible due to its deterministic nature. Secondly, it is very sensitive to the presence of outliers and measurement errors in data. Finally, it experiences dimensionality problems due to their slow convergence rates. In the next section, we expose the robust version of the nonparametric FDH model that we use in order to overcome these limitations.

3.2. The robust FDH approach

In order to improve the robustness of nonparametric methods Cazals et al. (2002) introduced the robust order-m estimation. This approach proposes to build up a partial frontier that envelops only $m \geq 1$ observations randomly drawn with replacement from the empirical sample of y_i such that $x_i \leq x$.. This procedure is repeated B times resulting in multiple measures ($\hat{\lambda}^1_{mi},..., \hat{\lambda}^B_{mi}$) using Equation (3) in each replication.

Finally, the order-m efficiency measure $\hat{\lambda}_m$ is computed as the simple average across the B efficiency scores. For acceptable m values, the efficiency scores will present values higher than unity, which indicates that students are inefficient, as outputs can be increased without modifying the level of inputs. However, in some cases we can observe $\hat{\lambda} < 1$, which means the evaluated student shows higher output level than the average m observations in its reference sample (Daraio and Simar, 2007). These students are called super-efficient. This is not possible in the traditional nonparametric framework whereby construction $\hat{\lambda} \geq 1$.

As we set before, the robust order-m FDH overcomes the main drawbacks of the original FHD model. Now, from the distribution of $\hat{\lambda}^1_{mi},..., \hat{\lambda}^B_{mi}$ we can make statistical inference (e.g. confidence intervals). As it does not include all the observations, it is less sensitive to outliers, extreme values or noise in the data. Moreover, Cazals et al. (2002) show that the convergence rate of this order-m estimator is comparable to parametric estimators, thus this estimator avoids the curse of dimensionality problem. As m increases, the expected order-m estimator tends to the FDH efficiency score $\hat{\lambda}_{FDH}$. Finally, this approach allows us to avoid the problem of bias that can arise when we compare groups of units on a different size (schools) since the mean level of efficiency generally depends on the existing number of students in each school (Zhang and Bartels, 1998). This problem can be reduced by using the same m parameter for every school, which implies assuming that the performance of every student is compared to the same number of units independently of the number of pupils included in the sample for each school. Following Daraio and Simar (2007) we set the value of m as the minimal level required to vary the proportion of super-efficient students only marginally with the size of m. In our application this value is equal to 40.

3.3 Metafrontier approach

In the educational context, the data have a hierarchical structure since pupils are nested within schools. To deal with this issue Silva-Portela and Thanassoulis (2001) and Thanassoulis and Silva-Portela (2002) suggest an approach to decompose the effect of school from students' inefficiency. Rooted on these ideas, the non-parametric literature developed the concept of 'metafrontiers' to deal with a hierarchical dataset to avoid biased results (see, for example, Battese and Rao, 2002; Battese et al 2004; O'Donnell et al, 2008). This approach measures the efficiency of units relative to separate best practice frontiers (local frontiers for each school) and allows us to decompose the performance of each student into a part attributable to the school and a part attributable to her skills.

Assuming we have K schools, each having their own technology and environmental factors, a metafrontier is defined as the boundary of the unrestricted technology set (including all students from all schools in the sample). The metafrontier (global frontier) can be represented by the technology set defined by:

$$\psi = \left\{ (x, y) \in \Re_{+}^{p+q} \;\middle|\; x \text{ can produce } y \right\} \tag{4}$$

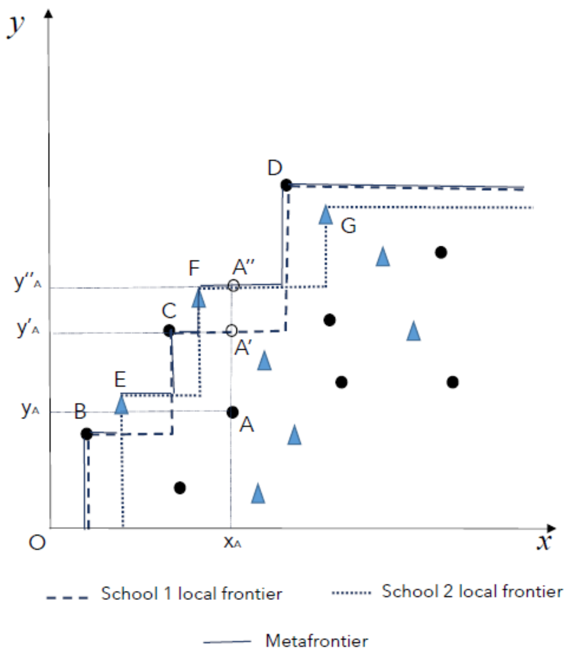
Separately, the local technology set for school k is defined as:

$$\psi^{k} = \left\{ (x_k, y_k) \in \Re_{+}^{p+q} \;\middle|\; x_k \text{ can produce } y_k \right\} \tag{5}$$

Then, following Thanassoulis and Silva-Portela (2002) the distance to the metafrontier is used to compute the *student-within-all-schools efficiency* or the overall efficiency (OE). The distance of the student to the school she attends is used to estimate the *student-within-school efficiency* (STE), which depends on her own effort. Finally, the distance separating the local and the metafrontier can be interpreted as the *school-within-all-schools efficiency* (SCE). Then, the overall pupil efficiency (OE) can be expressed as: OE = STE x SCE. We illustrate these concepts in Figure 3 in the single-input single-output framework.

We assume two schools: school 1 where students are represented by dots and school 2 where students are represented by triangles. The wider dotted line represents the local frontier for school 1, the thinner dotted line represents the local frontier of school 2 and the solid line represent the metafrontier (defined by the pooled sample). Consider again student A attending school 1. The *student-within-all-schools efficiency* corresponds to the distance $0y''_A/0y_A$, the *student-within-school efficiency* can be computed as the distance $0y'_A/0y_A$ and finally, the *school efficiency component* can be represented by the ratio $0y''_A/0y'_A$.

Figure 3 The metafrontier approach



3.4. Multi-level metafrontier approach

Since school resources and the environment where the school operates are heterogeneous, we need to extend the approach by Thanassoulis and Silva-Portela (2002) to include not only student data, but also additional variables representing school level characteristics (Thieme et al. 2013; Cordero et al. 2016).

Therefore, following the same procedure described in the previous section, we first decompose the school efficiency (SCE) into the resource endowment effect (REE) and the residual school efficiency effect (SCE2). Then we can decompose the overall pupil efficiency into three components: OE = STE x REE x SCE2. Second, public schools in Uruguay operate at quite heterogeneous communities and contexts. Particularly, in the most disadvantaged environments, learning processes might get more chaotic and disruptive and children might be more prone to fail (Grosskopf et al, 2001). As we think it is important to take these characteristics into consideration, we also isolate the effect of the environment efficiency endowment (EEE) from the school effect. Thus, we estimate a new global frontier (metafrontier 3) including the environment characteristics attributable to the context in which the school operates (EEE).

As a result, based on the sequential consideration of individual and school-level inputs we can expand the decomposition of the overall efficiency into four different effects: OE = STE x REE x EEE x SCE3. Now the residual part of inefficiency detected in students´ performance that can be attributed to other school factors can be identified as a residual school effect (SCE3) which represents the net impact of the school on efficiency.

It is important to note that REE and EEE are beyond the school's control. Resources in public schools are allocated by the education authorities and schools cannot move to a different district or influence (at least in the short term) the environment where they operate. If we do not consider these variables, the school effect (SCE1) could be biased, since we are implicitly assuming that all the schools are operating with the most favorable environment, which would not be real in many cases.

4. Data and variables

This study uses data on primary school pupils in Uruguay compiled in an evaluation called ARISTAS. ARISTAS is a comprehensive national survey which gathers the results of the first national assessment on schooling achievements led by the National Institute for Educational Evaluation in 2017 (INEEd, as its Spanish acronym for *Instituto Nacional de Evaluación Educativa*). A similar evaluation was also carried out for secondary education in 2018.

The database contains the results of standardized assessments of learning achievements that test whether students have effectively learnt what they were expected to according to the goals set by the official curricula in the fields of language and mathematics. Together with these data, ARISTAS also presents two additional sets of information: one relates to the socioeconomic and cultural characteristics of primary school students, their families and children's socio-emotional skills. The other refers to the type of school the students attend (private or public and which sort of public school), the school resources, the relationships among the teaching staff, the families and the community and other management issues. Information has been collected through questionnaires answered by pupils, parents, teachers and school principals. The survey has been applied to 3rd and 6th grade students at the national level, that is those in the middle and in the end of the primary schooling cycle. The dataset includes information about 247 private and public schools (13,4% and 86.6% of the total, respectively) and approximately 7 thousand students in the 3rd and 6th grade (the precise number of students varies around that figure, according to the number of pupils who effectively answered the different questionnaires).

Based on ARISTAS, the sample used in this paper just includes urban public schools. This election is grounded in the following reasons: first, it allows us concentrating in the challenges and shortcomings faced by the most important schooling provider in the country. Besides, we avoid any possible selection bias that may interfere in the efficiency analysis because the schools are not be totally comparable. Particularly, school biases might arise because public and private schools do not operate under the same rules and because pupils from higher socioeconomic status have a higher probability to self-select into private schools (see Section 2). Finally, we exclude Rural schools because they operate under a reduced scale and their pupil characteristics and resource endowments generally differ from urban schools.

Furthermore, we focus on pupils in their 6th grade (at the age of 11) instead of 3rd grade students (normally, aged 8). The decision hinges upon the belief that the older the child the more precisely the efficiency measure captures her effort and compromise with the learning process. Also, to build fair comparisons, we do not include pupils reported to have special needs or face particular physical or mental difficulties. As a result, our final sample comprises 144 schools and 4120 students with complete data from both Montevideo (the capital city) and urban areas in the rest of the country. To ensure enough variability, we constrain our assessment to schools with 15 students or more. Table 1 provides information on the distribution of the selected students and schools (by school type).

Table 1. 6th grade students by school-type in the sample

Type of school	Students	%	Schools	%
"Learning"	816	19.8	27	18.8
Practice	798	19.4	24	16.7
Full Time schs.	1359	33.0	54	37.5
Regular schools	1147	27.8	39	27.1
Total	4120	100	144	100

Note: Extended Time schools, though urban, are not included because their representation was just marginal in the original ARISTAS database.
Source: own computation based on ARISTAS database

We assess whether the students are making the most to convert their potential into the best possible performance by estimating a best practice frontier. Our preferred model to measure the “pupil within all schools- performance” includes two outputs and one input. The outputs are the tests scores obtained by students in reading and mathematics, standardized to a mean of 300 and a standard deviation of 50 points. In the case of inputs, following Thieme et al. (2013) and Cordero et al. (2016), we take a measure of the socioeconomic and cultural background of pupils. From the Coleman Report onward the literature has identified this condition as one of the most important predictors of educational achievements (Hanushek and Luque, 2003). The variable is based on an index of economic, social and cultural status computed by ARISTAS according to the information provided by parents and students. It has been created from 12 different items related to the student’s household expenditures and consumption. The indicator also includes information on the educational and cultural background of the student’s family (INEEd, 2018a).

In order to disaggregate the student from the school efficiency effect we resort to a multilevel model by sequentially considering two additional inputs at the school level. The related information proceeds from the responses of school principals. As will be seen, this input mix includes factors that are beyond the control of schools, but greatly condition their performance. We first consider the resource endowments or school resources the institutions provide to their pupils so that they can improve their academic achievements. This dimension is represented by two variables: one, summarizes data on the facilities and the infrastructure conditions at the school. We build it by merging data about the school availability of meeting rooms for teachers, gymnasium, sciences laboratory, library and adequate sewage, bathrooms, window glasses, electrical connections, tubing, walls, ceilings and curtain walls. Responses about all these items were adjusted so that the composite index expresses an increasing availability. The other variable refers to the "peer effects" measured as the average of socioeconomic and cultural status of students who attend the same school. It is expected that a child's school experience is strongly affected by the students with whom she shares the classroom or school (Hanushek et al., 2003). This, in turn, conditions the scope of the educational achievements the school can attain.

The environmental dimension is captured by two variables: crime and violence in the school neighborhood and learning motivations of the students (presumably associated to the extent of family involvement in the learning process). The index for crime and violence is built upon a Principal Components Analysis (PCA) on variables reflecting the school principal perception about the probability of violent episodes in the school's surroundings. These might include public disorders, violent arguments among neighbors or armed robberies.[1] Complementarily, to get an index on learning motivations we have proceeded in two steps. First, we applied a PCA on a group of 11 variables reflecting the school principal opinion about the attitude of pupils toward the learning process, theirs and their family's involvement in educational achievements, their health and previous abilities. The methodology produced two components that largely explained the variation in this set of indicators. One of them has been interpreted as measuring "learning motivations" because it comprises the presence of classroom difficulties in interactions, pupil's interest in learning and family's commitment with the learning process.[2] This measure as well as the one on crime and violence has been adapted to fulfil the isotonicity requirement, so that increasing the inputs would lead to maintain or increase outputs.

For socioeconomic and cultural indexes and the two measures comprised in environmental conditions the original variables presented negative values. Therefore, we rescaled them to be properly included into our nonparametric frontier models. Table 2 reports the descriptive statistics for the selected outputs and inputs. All inputs show positive and statistically significant correlations with the outputs (Table A-1 in the Appendix).

[1] ARISTAS provides a composite index measuring violence and crime. However, it was built to capture increasing violence rates. For that reason, we preferred to create a particular indicator in line with the isotonicity condition.

[2] Due to missing 63 missing cases, this variable has been imputed based on a formula that considers the average socioeconomic and cultural level of the school.

Table 2. Descriptive statistics for output and input variables

	Obs.	Mean	Std. Dev.	Min	Max
Outputs					
Mathematic scores	4,120	296.5	47.4	138.8	553.2
Language scores	4,120	299.0	48.0	121.7	468.6
Inputs					
Student level:					
Socioeconomic and cultural status of students	4,120	2.8	0.8	1.0	5.3
School level:					
School resources					
School facilities	4,120	6.3	2.3	1.0	10.0
Mean socioeconomic and cultural status at school	4,120	1.9	0.4	1.0	3.3
Environmental conditions					
Learning motivations	4,120	5.9	2.0	1.0	10.5
Crime and violence in school's surroundings	4,120	4.4	1.9	1.0	7.5

Source: own computation based on ARISTAS database

Table 3 reports the output and input variables for the different types of public schools. Regarding outputs, the average score both in mathematics and language is related to the socioeconomic characteristics prevailing in the school population. Accordingly, Learning and Full Time schools, which belong to unfavorable districts report lower tests performance than Regular and Practice schools (see Section 2). However, it is important to mention that top and bottom achievers can be found in all school types (INEEd, 2018a). These output values are related to the inputs available at each type of school. Thus, school facilities improve as schools are situated in better socioeconomic contexts as shown by the mean socioeconomic status of students at each institution. Notwithstanding, there seems to be a minimum endowment shared by all schools (the mean value across school types is above 5 in a scale that reaches 10). Finally, we note that crime and violence at school's surroundings and learning motivations reflect the worst condition in Learning followed by Full Time schools.

Table 3. Descriptive statistics for output and input variables by school type

	"Learning" sch.		Practice sch.		Full Time sch.		Regular sch.	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Outputs								
Mathematic scores	282	42.7	309	48.4	293	48.1	303	45.9
Language scores	285	46.4	311	47.3	294	47.2	306	47.4
Inputs								
Student level:								
Socioec. & cultural status of students	2.4	0.6	3.1	0.8	2.60	0.7	2.94	0.7
School level:								
School resources								
School facilities	5.5	2.1	6.5	2.4	7.3	1.8	5.7	2.5
Mean socioeconomic and cultural status at school	1.5	0.2	2.3	0.4	1.7	0.4	2.0	0.4
Environmental conditions								
Learning motivations	5.3	1.9	7.3	1.2	5.4	1.8	6.0	2.1
Crime and violence in school's surroundings	2.9	1.7	6.0	1.4	4.0	1.7	4.9	1.6

Source: own computation based on ARISTAS database

The information on the key variables of the analysis confirms a long lasting bias in the Uruguayan schooling system: those students and schools in the lowest end of the socioeconomic distribution are also the most disadvantaged in terms of teaching and learning conditions. This initial distribution of outputs and inputs allows recognizing that schools could be reproducing the schooling gaps more than compensating those students in more need. This study aims to learn whether this performance of students and schools represent the maximum they are able to achieve from the given resource allocation.

5. Results

5.1. Student and school effect decomposition

Table 4 shows the estimated scores and the differences among efficiency components for the sample comprised by 4120 public primary school students. As previously stated, efficiency scores higher than unity indicate the range of inefficiency, that is the extent to which both outputs could be expanded without changing the level of inputs. The mean value for the overall efficiency score is 1.192. This means that if all pupils perform as efficiently as the best pupils in the sample, the test scores attained may increase 19,2%. The ratio raises to 1.218 when we just consider only inefficient students. The quantile specification shows a considerable variation: 25% of the pupils who attain the best ratios (1st quartile) are almost 10% short of the potential score they might have obtained. Instead, at the other end of the distribution, students in the 3rd quartile are almost three times more inefficient than their peers in the 1st quartile.

Regarding the roots of the pupil's under-attainment, the mean value corresponding to the student efficiency component (STE) is 1.120. That is, on average, pupils attain tests scores 12% lower than they could be expected (the last column in Table 4 shows that this score remains the same when we only consider inefficient students). The top performing pupils (in the 1st quartile) are perfectly efficient while those at the bottom 25% would be able to expand their scores by 20%. The ratio of the standard deviation over the mean is roughly the same for the overall and the student efficiency ratio. Table 4 also reports that the mean value of the school efficiency component (level 1) is 1.064. This figure assumes that all schools operate at an optimal level of resources and environmental conditions. Therefore, if schools manage to improve their own performance and behave efficiently they would contribute to expand pupils' educational outcomes by an average of 6.4%.

Table 4. Decomposition of overall efficiency effect in three levels

Efficiency decomposition	Mean	SD.	Min	Max	1st quartile	Median	3rd quartile	Mean (inefficient)
Overall efficiency (OE)	1.192	0.166	0.708	2.023	1.09	1.18	1.30	1.218
Student efficiency (STE)	1.120	0.142	0.927	1.894	1.00	1.09	1.20	1.120
Level 1								
School efficiency (SCE1)	1.064	0.193	0.532	1.867	0.95	1.07	1.20	1.087
Level 2								
School resources eff. effect	1.027	0.033	0.867	1.421	1.01	1.02	1.03	1.028
School efficiency (SCE2)	1.037	0.187	0.529	1.802	0.92	1.04	1.17	1.058
Level 3								
Environm. conds. eff. effect	1.011	0.042	0.862	1.378	0.99	1.00	1.01	1.012
School efficiency (SCE3)	1.026	0.184	0.531	1.866	0.92	1.03	1.16	1.045

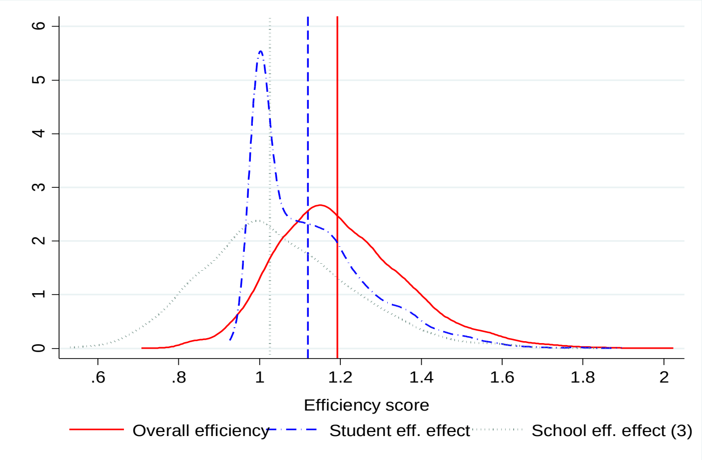
Results on STE and SCE1 in Table 4 show that the extent of the outputs that the students do not obtain due to their own effort is quite higher than that attached to the school they attend. This outcome is in line with previous findings in the literature (Cordero et al 2016; Thieme et al., 2013; Portela and Thanassoulis, 2002).

The school resources effect (level 2) accounts for the part of the school inefficiency justified by the availability of facilities and the composition of the population attending the school (peer effects) at each institution (REE). Following Table 4, it represents 42% of the average school efficiency effect (1.027 vs 1.064). This means that almost half of pupil underachievement related to school inefficiencies can be attributed to its inadequate resource availability (which comprises the underlying characteristics of the pupil's schoolmates). In the third decomposition, it is the school efficiency effect (level 3) that is split into the effect of the environmental conditions (EEE) and the final or net school efficiency effect (SCE3). The score which controls for the level of violence and crime in the school's surroundings and the learning motivations of students (EEE) accounts, on average, for 17% of the original school effect.

Finally, after taking into account the previous resource and context factors, the net efficiency effect of the school (SCE3) is 1.026. This means that on average, the part of the overall efficiency that depends on the net school component has been reduced to 40% (1.026 vs. 1.064). Had schools behaved more efficiently, they could have contributed, on average, to make students obtain 2.6% higher scores. The magnitude of the inefficiency gap indicates that schools' contribution to the student's attainment seems to be very close to their full potential.

The distribution of the baseline components of the overall efficiency is pictured in the kernel density estimations in Figure 4. The vertical lines indicate the mean value of each of the scores. Starting with the overall efficiency (OE) and following with the student (STE) and the net school effect (STE3), we observe that the curves shift to the left (median values are progressively lower). Moreover, the overall and the net school efficiency have flatter and unimodal distributions compared to the measure for student effect. The density for the student effect seems to be bimodal: it has a peak around 1 (perfect efficiency) and another over the mean (1.12). The figure shows a considerable accumulation of values at the right-side of the mean showing a quite wide range of highly inefficient pupils.

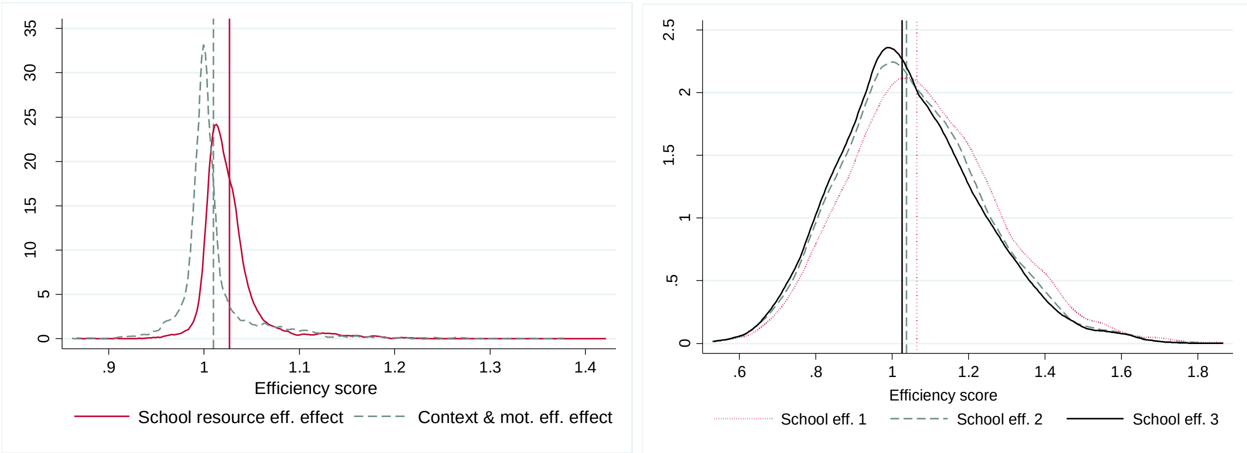
Figure 4. Kernel density estimation of overall, student and school efficiency effects



Note: vertical lines indicate the mean value of each component

Figure 5 pictures the density of the school resource efficiency (REE) and the environmental efficiency effects (EEE) in panel a, and the sequential densities corresponding to the three school level effects (panel b). Densities in (a) show that efficiency components at the school level contribute differently to the global effect. The school resource density is less tight than that corresponding to contextual factors and presents a higher average value. This validates that the range of school inefficiency due to the resource availability is wider than that explained also by the context in which it operates. On the other hand, densities in (b) show that the methodology effectively allows improving the precision of the diagnosis about the roots of school inefficiencies (school effect 1 to 3, respectively move to the left and values tend to accumulate closer to 1).

Figure 5 Kernel density estimation of school resource and context efficiency (a) and school efficiency components (b)



Note: vertical lines indicate the mean value of each component

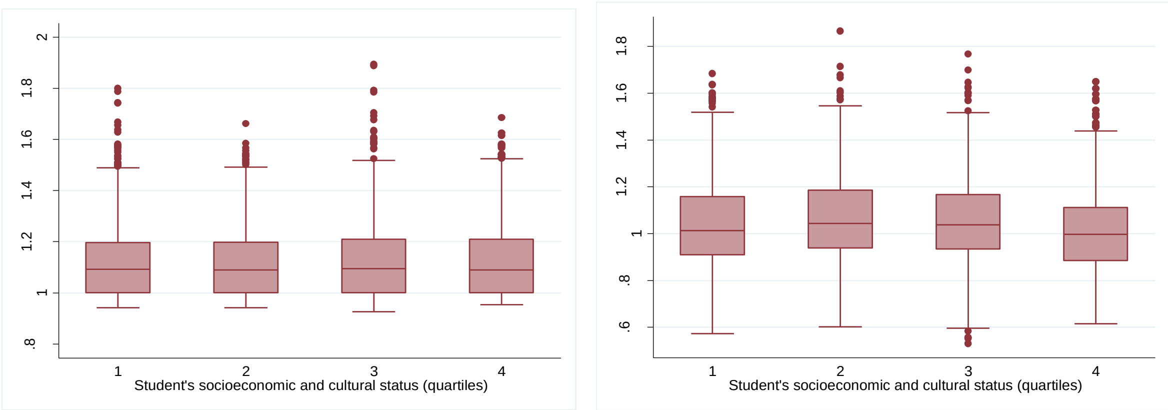
On the whole, the efficiency decomposition approach sheds light on the main sources of pupils' under-attainment. The analysis identifies where the main shortfalls lay but also provides estimates to appreciate the real importance of an efficiency improvement in terms of the original test performance. ARISTAS classifies student's proficiency at mathematics and language into 6 levels. Though there is an exhaustive description of the achievements included at each level, it is possible to establish that levels 1 to 3 comprise basic accomplishments while levels 4 to 6 describe moderate to full expertise in the corresponding area of knowledge (INEEd, 2018c).

5.2. Heterogonous student effects

We obtain an alternative approach to the efficiency scores over students by taking their distribution according to socioeconomic level and test scores. Box plots in Figure 4 (a) present slight divergences among the range of pupils' lack of effort by socioeconomic level. For 50% of pupils at each of the quantiles the efficiency is 1.09, the mean value reaches 1.12 and the super-efficient students are similarly distributed. The results are in contrast with the correlation between pupil test scores and socioeconomic status mentioned in Section 2 (Figure A-1). The only noteworthy result stems from the maximum efficiency scores which are particularly high in the 1st and 3rd quartiles.

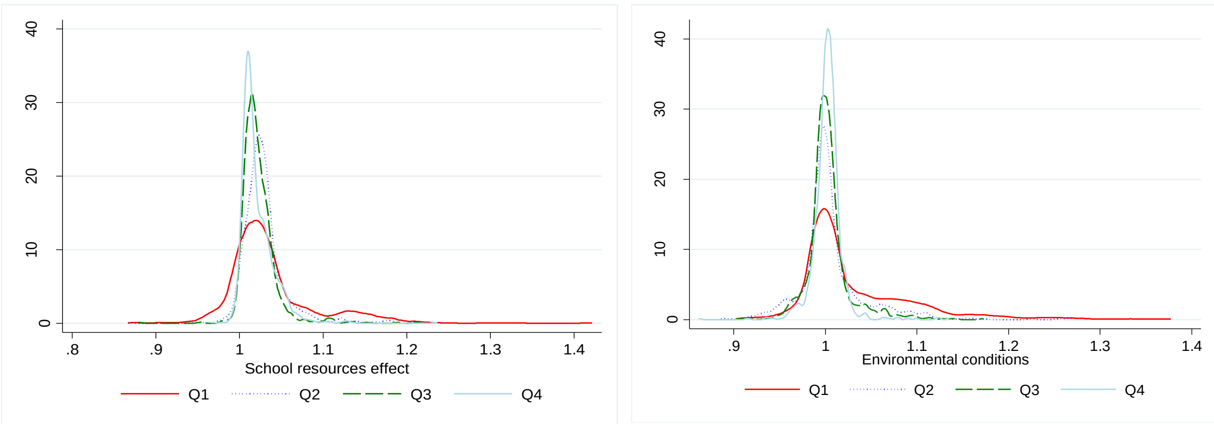
In addition, Figure 6 (b) tracks the pattern of the school efficiency effect computed at level 3 by pupil's socioeconomic condition. The figure shows that the median of values is very close to 1 with slight differences for those in the poorest and richest condition. The remaining divergences in school inefficiencies are negligible after controlling for resources and environmental conditions.

Figure 6 Distributions of student efficiency (a) and school efficiency effect (b) by students' socioeconomic level



However, a look at the distribution of resource and environment school effects for students at different socioeconomic quartiles show that school inefficiencies from both sources are particularly relevant for students in the 1st quartile (Figures 7a and 7b). For them, we observe the lowest concentration of efficiency scores around 1. This implies that those school inefficiencies related to resource availability and hard-to-manage environment conditions are particularly relevant to explain the school contribution to the efficiency of students at the lower end of the socioeconomic distribution.

Figure 7 Kernel density estimation of school resources (a) and environmental conditions efficiency effects (b) by student's socioeconomic status



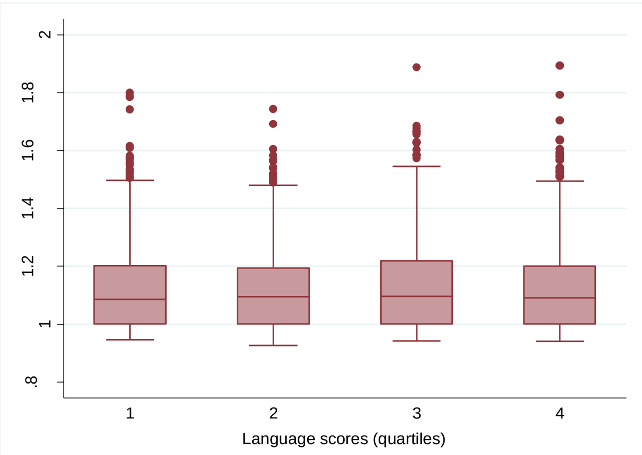
When we reproduce the previous analysis according to the distribution of language scores (in quartiles), the data emphasize the previous trends.[3] The student efficiency effect (Figure 6a) is still uniform over the groups of test performers. Therefore, the extent of the scores that the pupils are not achieving because of their lack of effort reaches, on average, 12%. The median student under-attainment reaches 9%. Even the range of super-efficient students are roughly the same across quartiles. However, we do find

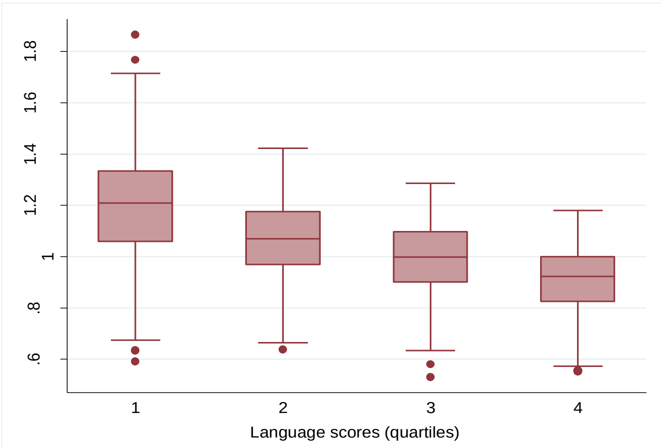
[3] For visual purposes the example refers to language scores as they present a higher variability. However, the analysis applied to math scores replicates the main conclusions.

variations in the output the students do not obtain due to school effects (not related to resources or environmental conditions).

Figure 8b reflects that schools do not seem to offset the poor attainment of the worst performing pupils. For the them, the median of the scope of improvement in total output which could be attributed to the school responsibility is 20%. Conversely, schools seem to be highly effective with the pupils who attain the highest test scores: the median school effect for students in the 2nd quartile is 1.07 while in the 3rd and 4th quartile inefficiencies are non-existent. Again, Figures A-2a and A-2b in the Appendix show that the lack of school facilities and environmental shortcomings have a wider effect on the school efficiency density of the children with the lowest average test scores (density functions are flatter than for the rest of students).

Figure 8 Distributions of student (a) and school efficiency effect (level 3) by language scores





These results suggest that the contribution of schools to the attainment of those pupils who have fewer academic competences from which to draw is limited compared to those who perform well. In other words, schools could provide far more aid to improve the output of the lowest performers, given the available resources. In part, this improvement should target school management difficulties which do not depend on resources or environmental conditions. But it should also consider inefficiencies attributable to inadequate resource endowments and environmental conditions, which have a visible effect on the "production capacity" of these students (Figure A-2a and A-2b). One possible explanation to this result might be related to insufficient teacher and support personnel. If teachers must devote more time to remedial learning and to cope with discipline problems just the abler pupils can take advantage of the teaching.

The efficiency decomposition may also provide insights on the differing performance of boys and girls. As before, we compare overall, student and school level efficiency effects for both groups. The results show that, on average, total inefficiencies are higher for boys than for girls (1.20 for boys and 1.18 for girls). However, the extent of the student efficiency effort to reach the optimum averages, again, around 12% for both groups (indeed, the differences are not statistically significant between genders).[4] There are variations in the school contribution to inefficiency: the score is 1.078 in the case of boys and 1.052 for girls at the first decomposition. After resource availability and context concerns are introduced, net school inefficiencies average 4.1% for boys and just 1.3% for girls. It is worth mentioning that school inefficiencies depending on environmental and context conditions seem to be more relevant to affect the school contribution to efficiency in the case of girls compared to boys. Results and their statistical significance are summarized in Table A-2.

5.3. School level analysis

In this section, we compute the efficiency decomposition for pupils attending the different school types included in the sample: i.e., Learning, Practice, Full Time and Regular schools (Table 5). Remember that Learning schools belong to the most disadvantaged contexts, followed by Full Time and Regular schools which are more diverse. Practice schools, generally, operate in more favorable socioeconomic contexts (Section 2). In addition to the descriptive approach, Table 6 includes the results of the Mann-Whitney tests on the equality of medians of each efficiency component taking two school-types at a time (Cordero et al., 2016). These estimates show whether the differences among school-types are statistically significant.[5]

According to the results, Learning schools present the highest mean (and median) overall inefficiency. They are followed by Full time, Regular and finally, Practice schools. Full time schools also exhibit the highest heterogeneity (they show the highest standard deviation as a share of the mean). Differences among school-types are statistically significant a 1% level. Regarding the individual level effort, the range of student inefficiencies is, again, roughly the same across school types (the ratio is, on average, 1.12). Therefore, no matter the type of institution, the maximum effort of individual students would lead them to improve their outputs by 12%. Indeed, test results indicate that the differences in this component are not statistically significant across school types.

[4] Differences between values for boys and girls are assessed according to the non-parametric Mann-Whitney.

[5] The null hypothesis for the test is that the medians of each of the variables are the same for each pair of school-types.

Table 5. Decomposition of overall efficiency effect in three levels by school type

School type	Learning schs.			Practice schs.			Full Time schs.			Regular schs.		
Efficiency decomposition	Mean	Median	SD.	Mean	Median	SD.	Mean	Median	SD.	Mean	Median	SD
Overall efficiency (OE)	1.23	1.22	0.169	1.162	1.156	0.149	1.202	1.187	0.173	1.175	1.170	0.158
Student efficiency (STE)	1.121	1.10	0.136	1.121	1.094	0.134	1.121	1.090	0.148	1.121	1.090	0.144
Level 1												
School efficiency (SCE1)	1.103	1.108	0.197	1.036	1.039	0.181	1.072	1.069	0.198	1.048	1.061	0.187
Level 2												
School resources effect	1.049	1.035	0.050	1.016	1.012	0.018	1.021	1.022	0.022	1.024	1.019	0.030
School efficiency (SCE2)	1.051	1.049	0.189	1.020	1.021	0.177	1.050	1.050	0.194	1.023	1.032	0.181
Level 3												
Context and educ. effect	1.019	1.001	0.052	1.006	1.003	0.017	1.012	1.000	0.053	1.005	1.000	0.029
School efficiency (SCE3)	1.031	1.019	0.184	1.014	1.016	0.175	1.038	1.032	0.193	1.018	1.026	0.179
Number of students		816			798			1359			1147	

Table 6. Mann-Whitney distribution tests by school type

Variable	Mann-Whitney test	Learning vs Practice schs.	Learning vs Full Time schs.	Learning vs Regular sch.	Practice vs Full time schs.	Practice vs Regular schs.	Full time vs Regular schs.
Overall efficiency (OE)	z-statistic	8.133	4.123	7.22	-4.886	-1.646	3.581
	p-value	0.0000	0.0000	0.0000	0.0000	0.0997	0.0003
Student efficiency (STE)	z-statistic	-0.892	-0.325	-0.492	0.575	0.384	-0.184
	p-value	0.3727	0.7453	0.6229	0.5652	0.701	0.8538
School effect (level 1)	z-statistic	6.899	3.725	6.002	-3.983	-1.556	2.698
	p-value	0.0000	0.0002	0.0000	0.0001	0.1198	0.007
School resources effect	z-statistic	20.174	15.396	14.674	-9.781	-10.438	0.002
	p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.9982
Cont. and educ. effect	z-statistic	1.217	4.483	4.811	3.687	4.486	-0.09
	p-value	0.2235	0.0000	0.0000	0.0000	0.0000	0.928
School efficiency (SCE3)	z-statistic	1.538	-0.879	1.254	-2.578	-0.442	2.372
	p-value	0.1241	0.3794	0.2098	0.0099	0.6583	0.0177

Note: tests on the statistical significance of the equality of medians of each distribution.

On the contrary, the first decomposition on the school level contribution to efficiency does differ among school types. Differences are statistical significant at 1% level, except for the comparison between Practice and Regular schools. The mean school effect (level 1) in Learning schools seems to be as important as the student effect to account for the overall efficiency (1.121 vs 1.103). A similar pattern appears at Full time schools (where the mean school effect reaches 1.072). As a result, in both school types a considerable part of the differences in the overall inefficiency depends on the school effect. The results are different for Practice and Regular schools. There, the contribution of the school effect to overall inefficiency is much lower and does not statically differ (the mean value of the school level component is 1.036 and 1.048, respectively).

This first insight can be adjusted when we include the influence of resources and environmental conditions on the school performance. After controlling for these factors, the school effect (level 3) has been visibly reduced compared to the first decomposition (level 1). The differences in this net school contribution to inefficiency at Learning schools are not statistically significant from the others. The same is true when comparing Practice and Regular schools. However, it is important to note that differences are statistically significant at conventional levels when Full time schools are compared to Practice and Regular schools. The median Full time institution is almost twice as inefficient as those of Practice and almost 20% more inefficient than Regular schools.

When focusing on the impact of resources, we find that inadequate school facilities and peer effects are particularly relevant to explain the efficiency performance by school-type. This component represents, on average, approximately half of the initial school level effect at all schools. The result is lower for Practice schools, as there, the effect explains 30% of the school efficiency score. Differences across schools are significant, except for the comparison between Full time and Regular cases, even though their initial resource endowment clearly favors the former (Table 3). Figure A-3 shows that the density function of this effect at Learning schools presents a lower accumulation of values around 1 and a flatter shape than the rest, pointing to a lower performance of this indicator over the entire distribution and a great variability.

Therefore, from a public policy perspective, the results underline the wide range of gains to be achieved if schools were to operate at their optimal level of resource availability. The results call the attention to the need of an adequate resource policy towards those already favored with comparatively higher resource endowments (as Full time schools) as well as to those which deal with the poorest socioeconomic conditions (Learning schools).

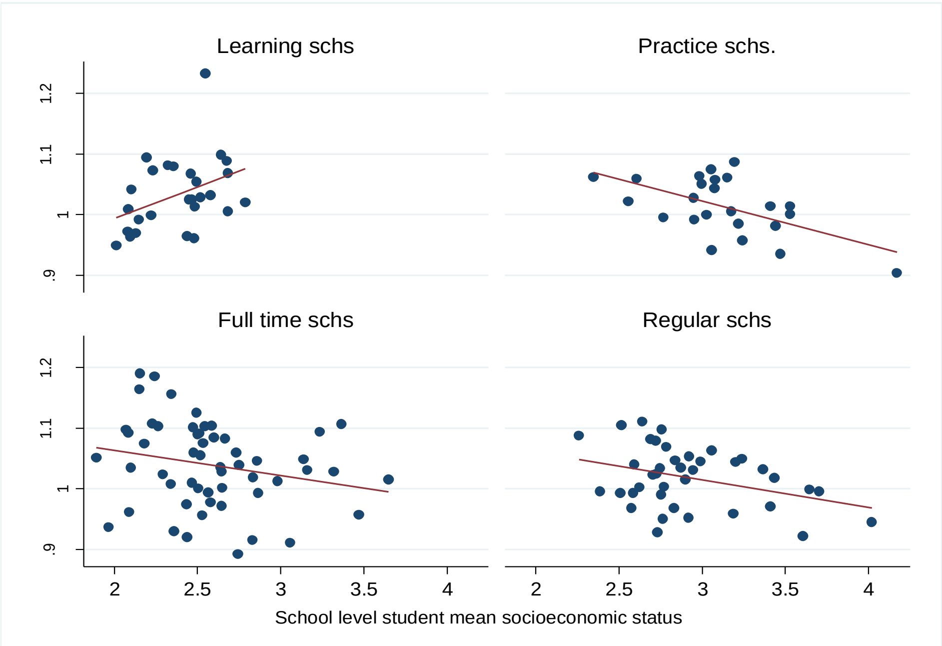
Alternatively, the components related to context and motivation show a comparatively modest influence on the school efficiency effect (taking averages, its relevance to explain the school contribution to efficiency ranges from 18% at Learning schools to 11% in the rest of cases). The value of this component is statistically different across school types except between Learning and Practice and Full time and Regular schools.

Finally, to get a picture of the behavior of the net school contribution to inefficiency across school-types, we adapt Silva-Portela and Thanassoulis' concept of "differential effectiveness" of schools to the discussion at the school-type level (Silva-Portela and

Thanassoulis, 2002). According to the authors, a school is "differentially effective" when it does not facilitate with uniform effectiveness the attainments of all its pupils. Hence, we aim to compare how, on average, the pure school inefficiency at different school types affects students according to the socioeconomic status and average test performance of the population of students. To visualize whether there is any trace of a differing pattern over groups of students, we build a school level data base and compute the average values of each of the efficiency effects by school types.

Hence, in Figure 9 we have plotted the net school effect against the average socioeconomic status of students at each school type. As a whole, the graph shows a negative correlation between the variables, except for Learning schools. This suggests that at school-types different from Learning, the higher the average socioeconomic level of pupils the better the school effectiveness. That is, in all but one school type, the education institution operates closer to the optimum level as its student population is richer. For these students, the efficiency performance tends to rely only on the pupil's effort. In turn, school do not compensate pupil's under-achievements for students at lower socioeconomic levels. The pattern is just the opposite at Learning schools. The school contribution to efficiency is higher for students at the bottom of the socioeconomic distribution. This feature fades for richer students. Therefore, this school type makes the best to obtain the highest possible results from the students under the worst conditions. In turn, there is room to make headway in the performance of the rest of students.

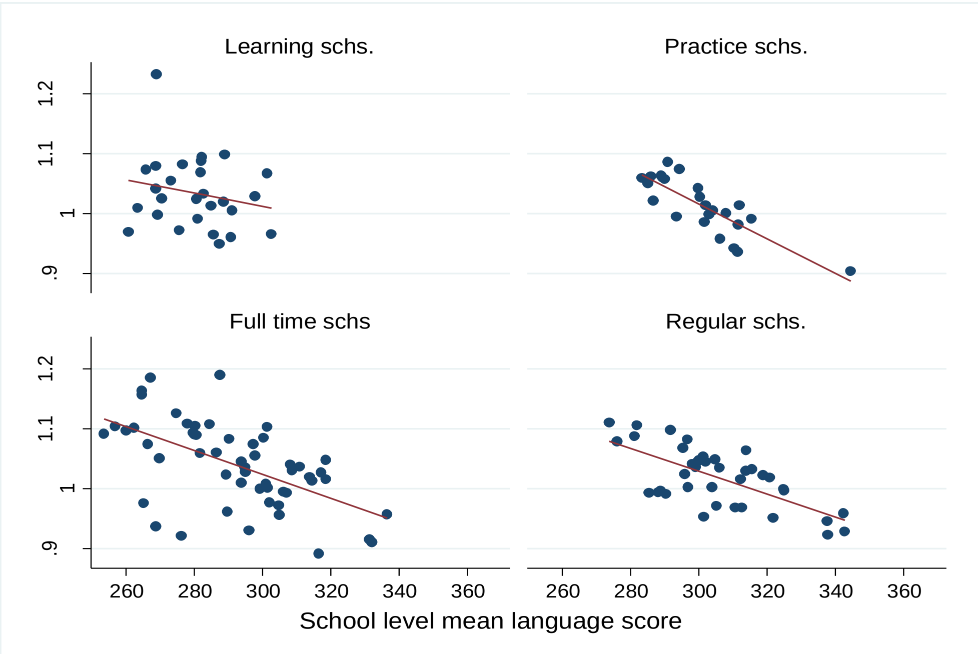
Figure 9. School efficiency effect (level 3) by pupils' average socioeconomic level across school types



Note: R^2 of the fitted linear regression is 0.15 at Learning schs., 0.32 at Practice schs, 0.05 at Full time schs. and 0.13 at Regular schs.

The trends in Figure 10 show that all schools roughly share the same orientation: the school efficiency effect decreases as the scoring tests raise. As a result, all school types seem to be more effective as their student population is more dedicated. The extent of the output students does not attain due to school effects is higher for the lowest performers. In this sense, no school seems to compensate the underperformance of those students in the lower levels of the tests rankings. The graphs suggest that this general trend is particularly strong at Full Time schools and not so remarkable at Learning schools.

Figure 10. School efficiency effect (level 3) by pupils' average language test score across school types



Note: R^2 of the fitted linear regression is 0.04 at Learning schs., 068 at Practice schs, 0.33 at Full time schs. and 0.46 at Regular schs.

6. Conclusions

Efficiency studies which aim to identify where resources might be used more effectively have increasingly been recognized as a useful tool to design school funding policies. However, the overall efficiency measure does not convey enough information to distinguish whether the main improvement efforts rely on the attitude of pupils or on the institutions they attend. In this paper, we apply a non-parametric meta-frontier approach to untangle student and school level inefficiencies. Besides, by considering a multi-level decomposition, the study also acknowledges that the efficiency result might be affected by differences in the input allocation across schools. Thus, the analysis sequentially adds information on resource availability and environment and motivations prevailing at the school level to effectively estimate a net school efficiency measure.

The empirical assessment resorts to the primary public education system in Uruguay. The schooling system in the country is characterized by the lack of adequate funding and high inequities in schooling performance depending on the socioeconomic background of students. We believe that efficiency estimates and their decomposition provide useful information to recognize the bottle necks attached to the performance of students and schools.

The study finds important efficiency gains to be achieved. On average, the overall efficiency improvement implied that more than 80% of students would be situated in the highest levels of proficiency in mathematics and language. Pupil's effort is the main responsible for the total inefficiency. The average value for this component is 12% and is homogenously distributed across socioeconomic levels, gender or school types. Notwithstanding, the scope of inefficiencies are considerable higher for the 25% of more inefficient students.

The range of output pupils do not attain due to school effects averages 6,4%. This magnitude considerably decreases after controlling for school resources and context factors: the mean school net inefficiency is 2,6%. However, pupils with the lowest test scores are those most affected by the larger net inefficiencies at the school level. Therefore, the school management is not compensating the poor performance of pupils.

The gap between the initial and net school effect points out to relevant school inefficiencies which do not depend on its management but on differences in input allocation. Particularly, the diminished peer effects and greater resource needs at the school level are, on average, more relevant than the context conditions to explain the school difficulties to make students reach their maximum output. Across school types, it is also resource availability the component which accounts for a larger share of school inefficiencies. There are some exceptions to this general rule because also context conditions matter to explain school inefficiencies affecting socioeconomically disadvantaged students and schools. Besides, context conditions particularly impact the school contribution to the efficiency performance in the case of girls.

Finally, we have compared the net school efficiency patterns for each school-type and find that there are traces of schools being "differentially effective". In general, they tend to be effective just with students of have the better attainments on the tests or with those of higher socioeconomic levels (though there is an exception with this latter finding at Learning schools). Hence, schools are likely to make even more difficult the performance of their more disadvantaged students.

According to the results, the efficiency decomposition avoids to unfairly assign a responsibility to schools for a range of inefficiencies they are not able to improve. However, it identifies relevant efficiency challenges to provide equitable learning opportunities for all students. These imply a revision of the optimal level of current school funding as well an enhanced school management, which should also imply new resource allocations.

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Appendix

Table A-1. Correlation between outputs and inputs

Inputs	Outputs	
	Mathematic scores	Language scores
Socioeconomic and cultural status of students	0.392	0.388
School facilities	0.049	0.043
Mean socioeconomic and cultural status at school	0.324	0.303
Crime and violence in school's surroundings	0.217	0.202
Learning motivations	0.126	0.116

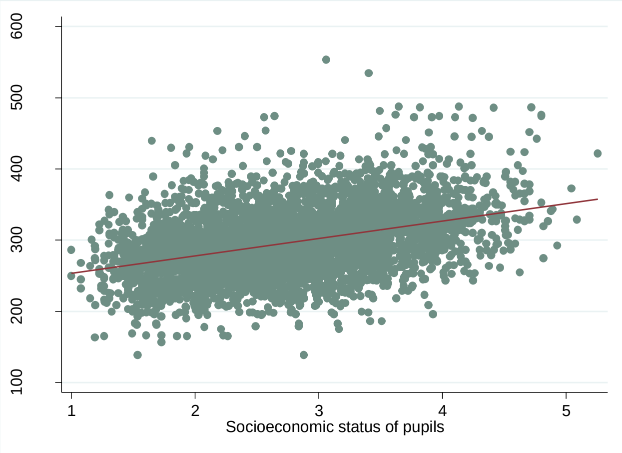
Note: all correlations are statistical significant at 1%.
Source: own computation based on ARISTAS database

Table A-2 Decomposition of overall efficiency effect in levels by sex

	Boys		Girls	
Efficiency decomposition	Mean	Std. Dev.	Mean	Std. Dev.
Overall efficiency (OE)	1.204*	0.171	1.182*	0.159
Student efficiency (STE)	1.116	0.138	1.124	0.145
Level 1				
School efficiency (SCE1)	1.078*	0.196	1.052*	0.190
Level 2				
School resources effect	1.027	0.138	1.027	0.033
School efficiency (SCE2)	1.050*	0.189	1.024*	0.184
Level 3				
Context and education concerns effect	1.009*	0.042	1.012*	0.042
School efficiency (SCE3)	1.041*	0.188	1.013*	0.180
Observations	1980		2140	

* 1% statistical significance of the Mann-Whitney tests.

Figure A-1. Correlation between test performance and socioeconomic context for students in the 6th grade of public primary education



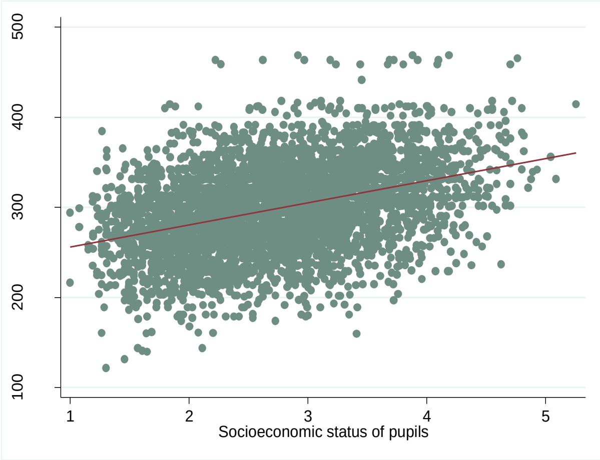
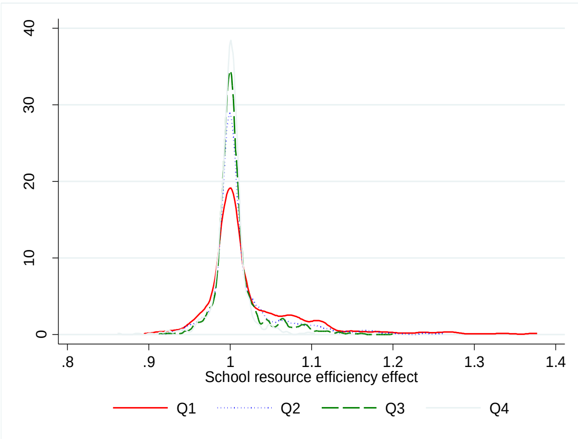


Figure A-2. School resources (a) and environmental conditions efficiency effects (b) by student's language scores



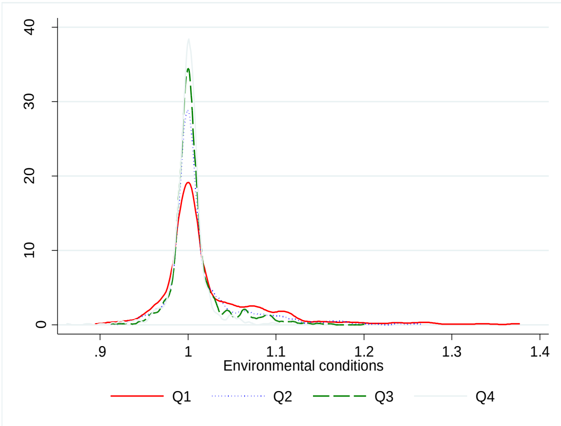


Figure A-3. School resources efficiency effects by school type